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OPTIMIZATION OF DISCRETE PID-ALGORITHMS STRUCTURE

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Comparative quantitative analysis of the quality of regulation automation systems functioning on the basis of different structure discrete PID-algorithms has been carried out. With the aim of solving the problem of the rational choice of the structure of discrete algorithms choosing discretization methods from the condition of maximum approximation of complex frequency characteristics (CFC) of the discrete analogue to CFC of continuous prototype has been suggested. Synthesis of compared continuous and digital systems by the method of multicriterial parametric optimization at different values of discretion period has been conducted. It has been shown that the use of discrete optimal structure PID-regulator allows to improve significantly the quality of the system compared to the serial discrete PID-regulator.

Keywords: *discretization, discrete PID-algorithm, automatic regulation systems*

Introduction. Statistic research shows that about 95% of modern automatic control-loops controllers function on the basis of different structure PID-algorithms. The structure of algorithms should be viewed in two aspects [1, 3]. Thus, by the way of formation of algorithm components with delay of the Defect we can subdivide them into (1) classic controller structure and (2) parallel structure, which are described by transfer function:

$$W_{pid}(s) = K_p\left[1 + \frac{1}{T_i s} + \frac{T_d s}{(T_d / N)s + 1}\right] = W_{pr} + W_i(s) + W_d(s), \tag{1}$$

$$W_{pid}(s) = K_p + \frac{1}{T_i s} + \frac{T_d s}{(T_d / N)s + 1} = W_{pr} + W_i(s) + W_d(s). \tag{2}$$

Where $W_{pr}, W_i(s), W_d(s)$ are proportional component, integral component and derivative component of the algorithm, the contents of which are obvious from fig. (1) and (2); $W_i(s)$ are setting parameters of the PID-controller, respectively, proportional gain and integral time constant T_d is derivative action time, N is a norming coefficient, setting the limit filter frequency of the first order at the differential component $F(s) = 1/[(T_d / N)s + 1]$. The value of this coefficient is usually fixed by controller manufacturers in the range of N = 1...33 (usually N = 10), though some companies provide the possibility of setting. The higher the value of N, the closer the PID-controller is to the idealized one.

Choice of structure of discrete PID-algorithm. As shown in [1], systems with parallel structure PID have worse quality parameters than systems with classic structure PID algorithm, K_p, T_i, T_d, are setting parameters of the PID-controller, N is a norming coefficient.

The structure of discrete analogues of continuous algorithms (1) and (2) also depends on the accepted methods of discretization of their components. With this aim the following expressions derived from the simplest methods of numeric integration can be used: Forward Euler, Backward Euler and Trapezoidal method. Combining these methods for discretization of PID-algorithm components we can realize its nine discrete analogues. Namely, such possibility is provided in MATLAB (R2010b) for modeling of discrete PID controllers. But there are no specific recommendations for using this option. Thus, the task of parametric synthesis of the system remains undefined.

As the structure of the algorithm influences the characteristics of the systems based on it, a problem of the rational choice of discrete algorithms structure arises. Some results of the research of this problem are covered in [1, 2]. Based on the analysis of algorithm (1) components' features it was defined that the maximum approximation of complex frequency characteristics (CFC) of its discrete analogue to CFC of a continuous prototype can be provided with integral component discretization by backward Euler, and derivative component discretization by trapezoidal method. Thus, an optimal discrete PID-algorithm should be described by the following function

$$W_{pid}(z) = K_p \left[1 + \frac{T_0}{T_i} \cdot \frac{1}{\left(1 - z^{-1}\right)} + \frac{\dfrac{2T_d}{2\left(T_d / N\right) + T_0} \cdot \left(1 - z^{-1}\right)}{1 - \dfrac{2\left(T_d / N\right) - T_0}{2\left(T_d / N\right) + T_0} \cdot z^{-1}} \right]. \tag{3}$$

Still, we have to note that in controllers produced by *Siemens*, *Schneider Electric* and some other producers the realization of discrete PID-algorithm is provided where integral component of continuous prototype is discretized by trapezoidal method, and derivative component – by backward Euler method. As a result, the discrete PID-algorithm, which can be called serial, can be shown the following way:

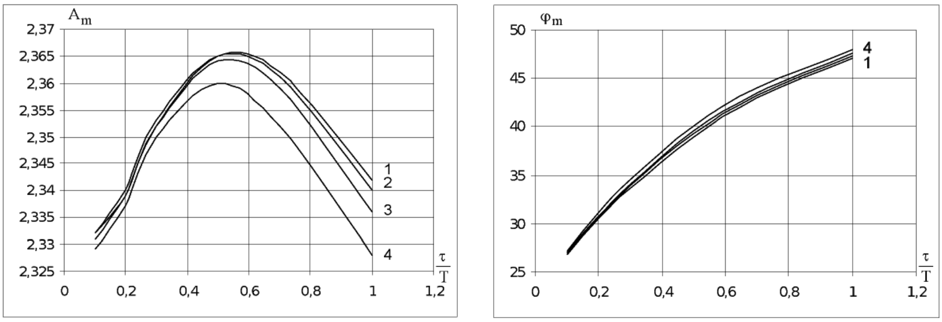
$$W_{pids}(z) = K_p \left[1 + \frac{T_0}{2T_i} \cdot \frac{1 + z^{-1}}{1 - z^{-1}} + \frac{\dfrac{T_d}{\left(T_d / N\right) + T_0} \cdot \left(1 - z^{-1}\right)}{1 - \dfrac{T_d / N}{\left(T_d / N\right) + T_0} \cdot z^{-1}} \right]. \tag{4}$$

Such an approach does not correspond to the recommendations in [1, 2]. Besides, we need additional arguments from more general positions in the form of a quantitative comparative analysis of characteristics of systems based on different structure algorithms.

Research tasks. The aim of this work is comparative quantitative analysis of the quality of systems working on the basis of algorithms (3) and (4). As an example the systems with process were taken. Process model is described by process variable delay τ and time constant T:

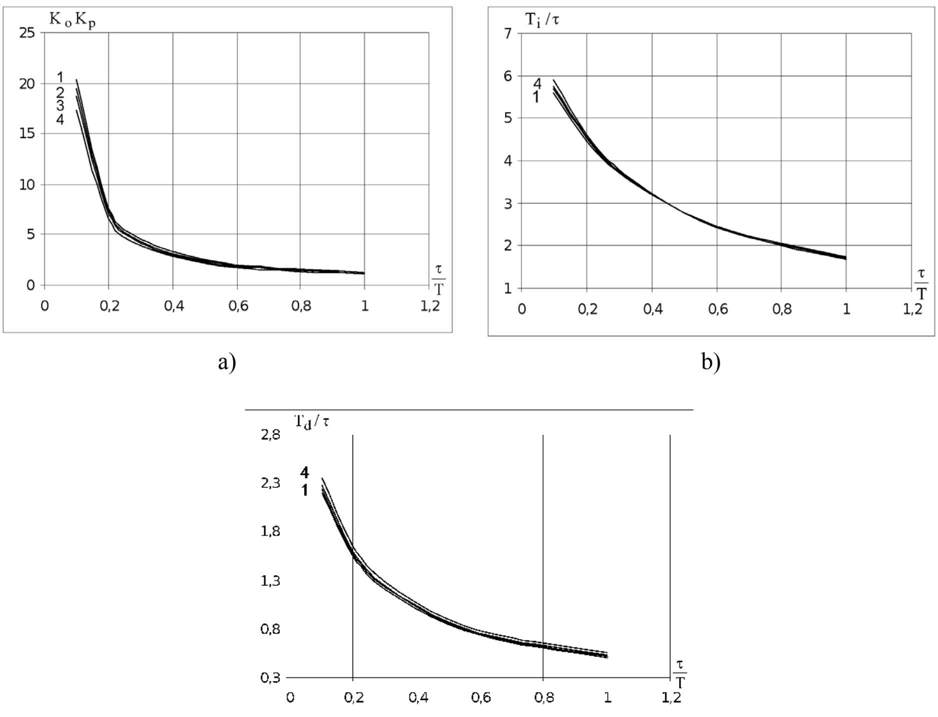
$$W_0(s) = \frac{K_0 e^{-\tau s}}{(Ts + 1)^2}, \quad K_0 = 1.$$

(5) **Systems quality comparative assessment.** The synthesis of continuous and digital systems was carried out by the Multicriteria Parametric Optimization method [1, 4] at the different values of Sample Time T_0. The main results of systems' comparative analysis can be found below in the form of tables (table 1, 2) and graphs (fig. 1 - fig.4).



a) b)

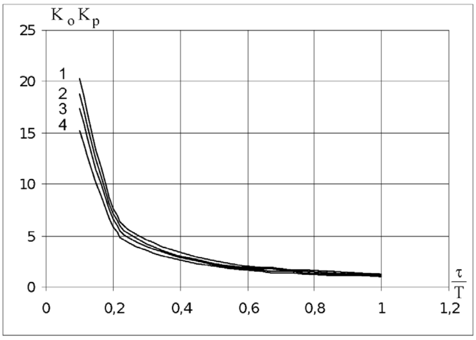
Fig.1. Stability indices of the system with continuous and optimal digital PID-controllers and an object model $W_0 = K_0 e^{-\tau s}/(Ts+1)^2$: 1 – continuous system; 2 – $T_0/\tau = 0{,}05$ 3 – $T_0/\tau = 0{,}1$; 4 – $T_0/\tau = 0{,}2$; a – gain margin A_m; b – phase margin φ_m^o



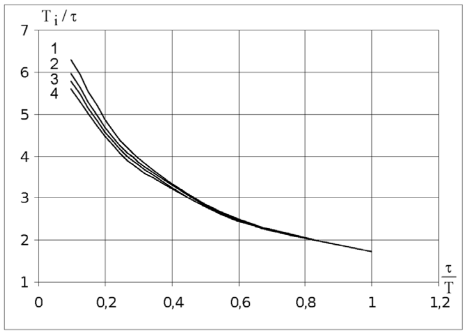
a) b)

c)

Fig.2. Parameters of classic continuous and optimal digital PID-controllers for the system with the object model $W_0 = K_0 e^{-\tau s}/(Ts+1)^2$: 1 – continuous system; 2 – $T_0/\tau = 0{,}05$; 3 – $T_0/\tau = 0{,}1$; 4 – $T_0/\tau = 0{,}2$



a)



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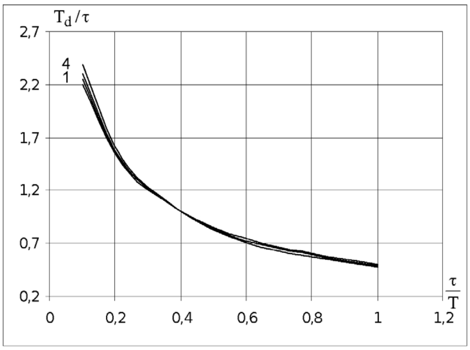
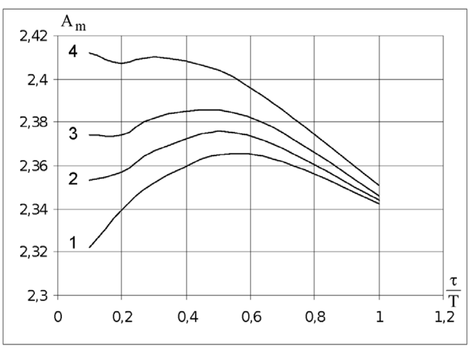
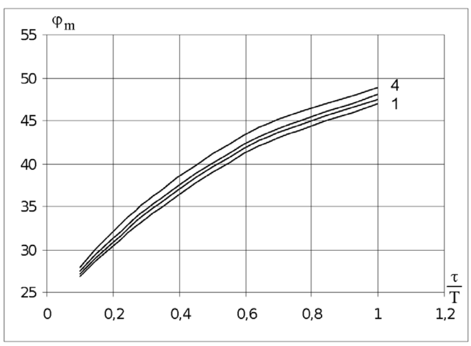


c)

Fig.3. Parameters of classic continuous and serial digital PID-controllers for the system with the object model $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$: 1 – continuous system; 2 – $T_0/\tau = 0{,}05$; 3 – $T_0 / \tau = 0{,}1$; 4 – $T_0 / \tau = 0{,}2$



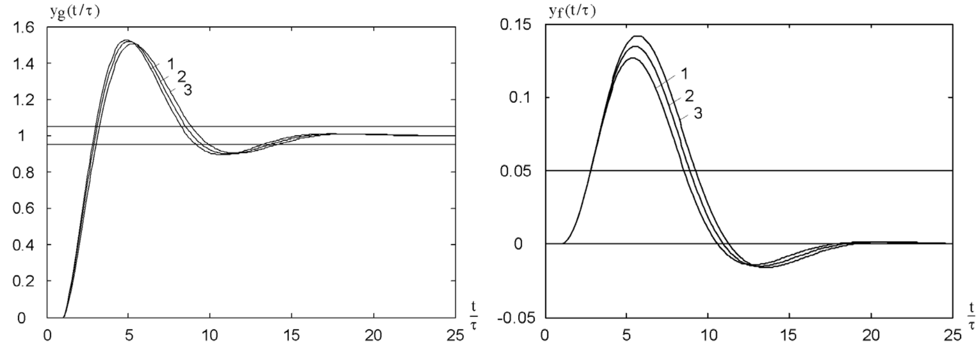
a)



b)

Fig.4. Stability indices of the system with continuous and serial digital PID-controllers and the model of the object $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$: 1 – continuous system; 2 – $T_0/\tau = 0{,}05$; 3 – $T_0 / \tau = 0{,}1$; 4 – $T_0 / \tau = 0{,}2$

For example, fig.5 shows transitory processes in systems with continuous and digital controllers of different structure, and table 1 shows relative values of their integral square errors (ISE).



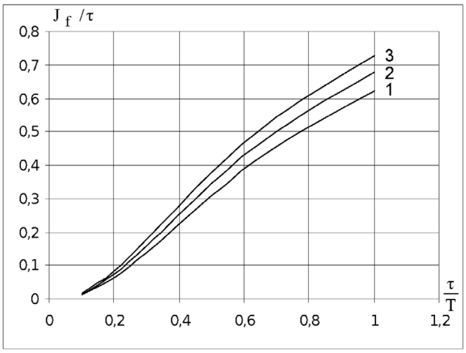
a) – $T_0/\tau = 0{,}2$ b) – $T_0/\tau = 0{,}2$

Fig.5. Processes of performing the task (a) and perturbation compensation (b) in systems with continuous (1), optimal (2) and serial (3) digital PID-controllers and the object model $W_0 = K_0 e^{-\tau s}/(Ts+1)^2$ at $T_0/\tau = 0{,}1$

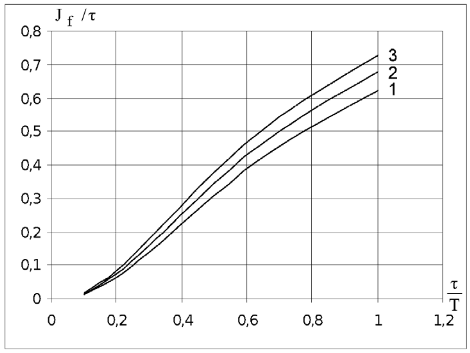
Table 1

Integral square processes analysis in systems with the object model $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$ and different controller types

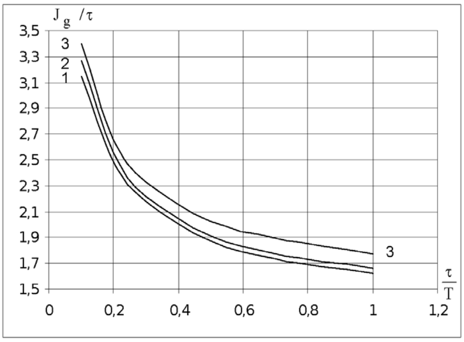
$\frac{\tau}{T}$	System with continuous	PID-controller	$\frac{T_0}{\tau}$	System with optimal digital PID-controller		System with serial digital PID-controller	
	J_g/τ	J_f/τ		J_{g1}/τ	J_{f1}/τ	J_{g2}/τ	J_{f2}/τ
0,1	3,146	0,0113	0,1	3,207	0,0136	3,274	0,0161
0,2	2,492	0,0612		2,526	0,0716	2,581	0,0822
0,3	2,172	0,1369		2,198	0,1572	2,252	0,1772
0,5	1,873	0,3055		1,894	0,3424	1,951	0,377
0,7	1,736	0,4548		1,757	0,5021	1,813	0,5453
1,0	1,626	0,6239		1,644	0,6789	1,70	0,7274
0,1	3,146	0,0113	0,2	3,265	0,0162	3,394	0,022
0,2	2,492	0,0612		2,558	0,0827	2,664	0,1065
0,3	2,172	0,1369		2,222	0,1782	2,328	0,2218
0,5	1,873	0,3055		1,916	0,3793	2,025	0,452
0,7	1,736	0,4548		1,776	0,5484	1,896	0,6368
1,0	1,626	0,6239		1,661	0,7322	1,768	0,8293



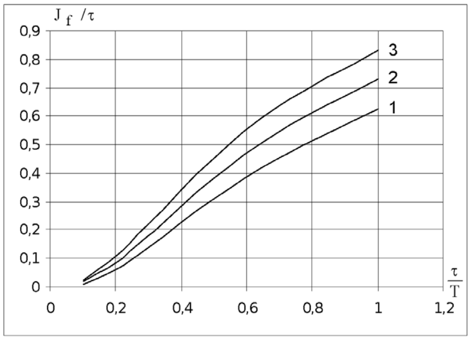
a) for digital systems at $T_0/\tau = 0{,}1$



b) for digital systems at $T_0/\tau = 0{,}1$



c) for digital systems at $T_0/\tau = 0{,}2$



d) for digital systems at $T_0/\tau = 0{,}2$

Fig.6. Integral square assessment of transitory processes in systems with the object model $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$ and different controller types: 1 – CLC with continuous classic PID controller; 2 – with digital optimal PID controller; 3 – with digital serial PID controller

The research shows that the quality of transitory processes by their integral square assessment in systems with discrete regulators compared with continuous system gets worse. The degree of this decrease for the processes of performing the task and perturbation compensation can be assessed calculating the deviation δ_g and δ_f by the expressions:

$$\delta_{g1} = \frac{J_g/\tau - J_{g1}/\tau}{J_g/\tau} \cdot 100\%\,; \quad \delta_{g2} = \frac{J_g/\tau - J_{g2}/\tau}{J_g/\tau} \cdot 100\%\,; \tag{6}$$

$$\delta_{f1} = \frac{J_f/\tau - J_{f1}/\tau}{J_f/\tau} \cdot 100\%\,; \quad \delta_{f2} = \frac{J_f/\tau - J_{f2}/\tau}{J_f/\tau} \cdot 100\%\,. \tag{7}$$

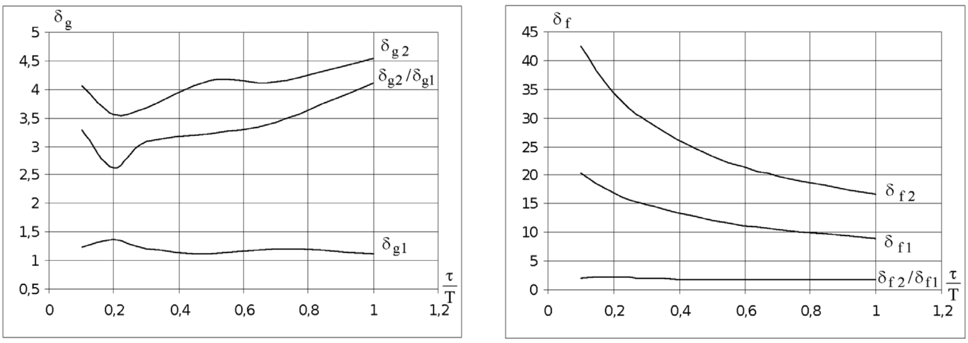
The results of calculations by these expressions are shown in table 2 and on the graphs (Fig.7). We have to note that the values of deviations δ_g and δ_f have the minus sign, which shows the decrease of quality in systems with digital controllers compared with continuous systems, but it is missed out for the sake of simplicity.

Table 2

Relative deviations of integral square assessments of transitory processes in digital systems with the object model $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$

$\frac{T_0}{\tau}$	$\frac{\tau}{T}$	System with digital optimal PID-controller		System with continuous digital PID-controller		Deviation correlation	
		δ_{g1}	δ_{f1}	δ_{g2}	δ_{f2}	δ_{g2}/δ_{g1}	δ_{f2}/δ_{f1}
0,1	0,1	1,239	20,35	4,069	42,48	3,284	2,088
	0,2	1,364	16,99	3,571	34,314	2,618	2,20
	0,3	1,197	14,83	3,683	29,437	3,077	1,985
	0,5	1,121	12,078	4,164	23,404	3,214	1,938
	0,7	1,21	10,40	4,135	19,90	3,417	1,913
	1,0	1,107	8,815	4,551	16,59	4,111	1,882

For making it more visual, the data in table 2 are presented in the form of a graph (fig.7)



a) b)

Fig. 7. Relative deviations of integral square assessment of transitory processes in digital systems with PID-controllers and the object model $W_0 = K_0 e^{-\tau s} / (Ts + 1)^2$

Conclusions. The research shows that the quality of transitory processes by their integral square assessment in systems with discrete regulators compared with continuous system gets slightly worse, but the best approximation is achieved at the optimal structure of digital controller. The degree of this approximation compared to the alternative variant is much better while using the controller with the optimal structure. The data from table 2 and fig.7 show that the use of optimal discrete PID controller gives the opportunity to improve the dynamic accuracy of processes of performing the task depending on τ / T of the object approximately 3 times, and the quality of perturbation compensation – two times compared with the serial discrete PID controller. Thus, the quality gain is substantial.

If we take into account the fact that program realization of the optimal structure of a discrete controller does not present any difficulty, it means that for developing high quality systems the issue of discrete PID-controller choice is solved clearly and effectively. A similar conclusion is true for systems with other object models as well.

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ОПТИМІЗАЦІЯ СТРУКТУРИ ДИСКРЕТНИХ ПІД-АЛГОРИТМІВ

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Виконано порівняльний кількісний аналіз якості автоматичних систем регулювання, що функціонують на основі дискретних ПІД-алгоритмів різної структури. З метою вирішення проблеми раціонального вибору структури дискретних алгоритмів запропоновано обирати способи дискретизації із умови максимального наближення комплексної частотної характеристики (КЧХ) дискретного аналога до КЧХ неперервного прототипу. Проведено синтез порівнюваних неперервної та цифрових систем за методом багатокритеріальної параметричної оптимізації при різних значеннях періоду дискретності. Показано, що застосування дискретного ПІД-регулятора оптимальної структури дає можливість суттєво покращити якість системи порівняно з серійним дискретним ПІД-регулятором.

Ключові слова: *дискретизація, дискретний ПІД-алгоритм, автоматичні системи регулювання.*

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