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MATHEMATICAL MODEL OF "BLACK BOX" OPERATING SYSTEM

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Mathematical model of system functioning as example of «black box» model is considered in this article. Parameters of some set are inputs of this model, which need to be defined and compared with existing ones. Outputs are information about the state of the object under study based on parameters of some set, which can be represented as reports, graphs, documents. There is no focus on the internal structure of the system by using the «black box» model. Mathematical model of the system operation to determine the level of diesel fuel quality in the tank of car is considered as an example. The information about the appropriate level of fuel quality, which has been sent to the consumer's mobile phone, is considered as an example. There is an opportunity to influence the inputs of the system in different ways during system operation and analyze its reaction on the corresponding input actions. Herewith reaction level of input actions is fundamentally related to the variety states of the system outputs.

Keywords: *mathematical model, system, black box, method, set, quality indicators, input, output.*

Formulation of the problem. «Black box» is meant to be a material system (object, process, occurrence), in relation to the internal organization, structure and behavior of which the observer does not have information, but he can manipulate by the system as a whole through its inputs and register its reactions through outputs. The research of the «black box» is as follows. An observer acts on the box through its inputs and receives information on its outputs. The observer and the «black box» form a system with feedback. Thus, the observer is able to analyze the so-called test protocol (as a chronological sequence of pairs «state of entry; state of exit») in order to find patterns in the behavior of the black box [1].

The name «black box» clearly emphasizes the complete lack of knowledge about the internal components of the «box». In this model, only the output and input connections of the system with the environment are specified [2].

The concept of «black box» was proposed by Ross Ashby, and allows us to study the behavior of systems, that is their reactions on various external influences, and at the same time to abstract from their internal structure. Thus, the system is studied not as a set of interconnected elements, but as something whole, interacts with the environment at its inputs and outputs. The «black box» model is used in various situations [3]. It is not only very useful, but sometimes a unique model for systems research. It can be applied to any field of knowledge of science and technology

(Fig. 1). That is, achieving certain knowledge (inputs) gain certain skills (outputs), without focusing much on the internal structure of the system. It is important to determine what you need to set on the input to the system, and what should be on the output from it, and it does not matter – what is inside the system.

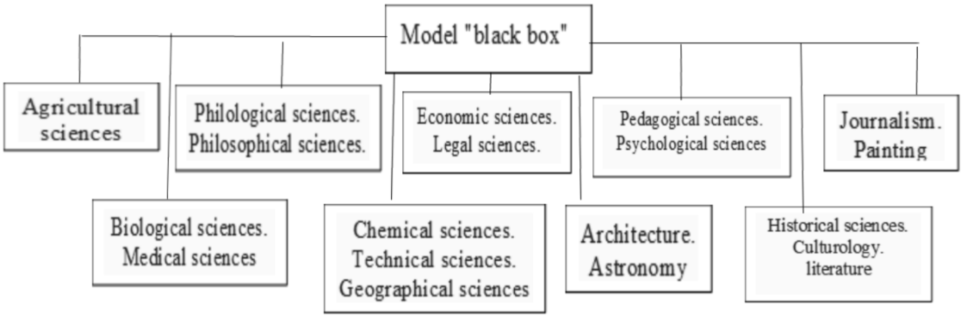

Fig.1. Fields of knowledge of science and technology

Such models are often dealt with in everyday life: for example, to drive a car or work on a computer, it is not necessary to know their internal structure. We face with usage of a «black box» all the time, switching the TV, heating food in the microwave, planting seeds for seedlings, and so on. For example, the doctor examining the patient asks him a series of test questions (inputs) and basis of the answers (outputs) and his own experience makes certain conclusions about the diagnosis. When taking an analgin tablet, it is not necessary to know the composition of the tablet itself and to present the mechanism of action of its components on the body, but it is important that the headache passes. By studying a cell in biology, we find out what comes into it and what comes out. Such knowledge already says a lot about the functions of the cell and its role in the body. Our knowledge of human organs: heart, lungs, kidneys, etc. – described by the model «black box». Knowledge of the cell nucleus, mitochondria and other cell forms is sufficient to treat many diseases. Changes in ecological systems under the influence of anthropogenic activity can also be described by this model, observing what level of load leads to imbalance in ecosystems and irreversible changes [4].

This model is used in the study of human psychology, when studying the methods of action on human consciousness and the consequences of this action. Forensic scientists also use the «black box» model. Research on the so-called «lie detector» is performed at the level of the «black box» model: input are words, output are changes in respiratory rhythm, pressure, electrical impulses of the brain, changes in skin conductivity, sweating, and so on. It is clear that the use of this model allows us to study its behavior.

Analysis of recent research. At first glance, this model is quite simple. There are many complex things hidden behind this simplicity. The disadvantage is the complexity of building a model of the type of «black box», which is that it is the basis for further study of the system and the development of specific proposals for practical activities. At the first stage of the system analysis, you can select a lot of input values. Then the system will turn from simple to complex, large, and it is very difficult to study. Or choose an insufficient number of input values and the system becomes simplified, studying it will not give a practical result.

If the model is used for the purposes of developing a specific project, for certain practical recommendations, then the errors of modeling and analysis in the initial stages are difficult to correct later. For example, the automotive industry. In the car model at the beginning of its industrial production, no one took into account the exhaust gases and thermal radiation. This has led to smog, which has affected people in large cities, to the depletion of the Earth's ozone layer, to global warming. These phenomena could be avoided if the initial values were initially included in the model and the automotive industry took them into account. Now it is too difficult to correct these consequences [4].

If the input and output parameters of the studied system are known, it means that quite a lot is already known about this system. Thus, the model of the «black box» system should be used in cases where the purpose of the study is to determine the outcome of the system without taking into account the internal structure of the system.

Purpose of the article. Construction of a mathematical model of the result of the system functioning using the model «black box» using only the inputs and outputs, without focusing much on the internal structure of the system.

Model of the black box

The simplest model for a reliable result of the system is so-called «black box» model (Fig. 2), in which the emphasis is on the purpose and behavior of the system, and its structure is only indirect information displayed in relation to the environment [5]. Connections with the environment that go into the system (inputs) make it possible to influence it, use it as a means, and connections that come from the system (outputs) are the results of its operation, which either affect changes in the environment, or consumed externally.

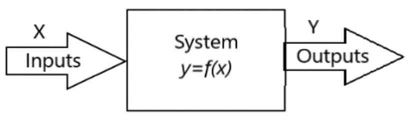

Fig.2. Black box model

In practice, the black box is considered to be the objects of research, the internal structure of which is unknown or not taken into account. Sometimes a meaningful description of the inputs and outputs of the system is enough. Sometimes presenting a system as a «black box» is the only way to study the system.

Presentation of the main research material. The method of building a mathematical model for a reliable result of the system using the «black box» model is to find the relationships between the inputs and outputs of the system. By observing the inputs and outputs of such a system, that is having observation vectors $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_m)$, it is possible to achieve such a level of knowledge about its functional properties that will allow to predict changes in the output components at any change of inputs, so you can find the mapping *f*:

$$X \to Y . \quad (1)$$

The mathematical model of the functioning result of the system by the method of «black box» is a procedure for determining the input and output variables. Let input X and output Y be subject to change regardless of their physical nature. Assigning a series of values to the input variable X_i, we obtain a series of values of another variable Y_i, so a series of observations (Table 1).

Table 1

The values of the parameters of the variables *X* and *Y*

X	X_1	X_2	....	X_n
Y	Y_1	Y_2	....	Y_n

Thus, the primary data of any study of the «black box» are formed only from a sequence of values of the vector with two components – input and output. Having obtained a fairly long series of observations, the search for patterns in the behavior of the box begins, the search for the recurrence of such behavior. If the system is not deterministic, so the input-output transformation is not unambiguous, you can go by one of two ways: 1st way – to change the set of inputs and outputs, taking into account more variables and increasing the number of observations, and then find out whether the new system is deterministic; The second way is to abandon the search for strict determinism and try to find a statistical pattern.

During system operation research, it is possible to analyze its reaction to the corresponding input actions by influencing the input of the system in different ways. The level of different input actions is fundamentally related to the diversity of the system output states. If the system responds in an unpredictable manner to each new combination of input actions, the test must be continued. If on the basis of the received information the system which exactly repeats behavior of the investigated can be constructed, the task of result of functioning of system can be considered solved [6].

To build a mathematical model, it is necessary to use parametric-oriented design of information systems to obtain the result of system operation. When designing information systems based on parametric configuration of the application package (AP), the latter is considered as «black box» (Fig. 3). Parametric flows are inputs of the AP, and the output is the result of the packet operation. AP includes the following blocks: operation, parameter processing, adaptation.

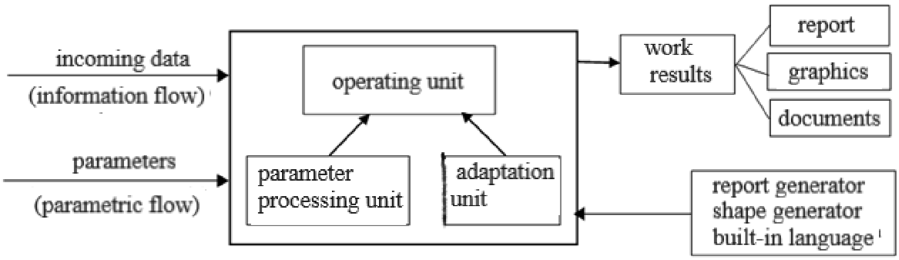

Fig. 3 Representation of AP in the form of a «black box»

The information stream is the input data that is processed and necessary to obtain the results of the package. The input data for the package operation can be presented in the form of various documents. The results of the package can be presented in the form of graphs, electronic documents, which can be accumulated or sent to the internal environment.

The operating unit processes the input data and generates the results of the package. Graphically, the unit of operation is a tree of software modules that automate data processing functions.

Parametric stream is represented as information that is needed to configure the package for specific operating conditions. The parametric stream includes information that is specified once when installing this package. By changing the settings, you can turn on and off any modules or affect the mode of their operation. For the client-server architecture, the parametric flow describes the users and their levels of access to the software modules and to the package as a whole.

The parameter processing unit is a set of special modules for interpreting parameter values. In part, the parameter processing unit transfers user settings directly to the application and to the database being used. The AP configuration that is performed allows you to use it for a wide class of control objects.

The adaptation unit interacts with the operation unit and can add modules or modify them. The need to use the adaptation unit is related to the need to refine the software modules of the AP under the influence of external operating conditions. Therefore, the AP includes tools for adapting existing standard design solutions.

Tools, which are available to a qualified user, are used: generators of information systems programs based on different programming languages; macro languages of design and configuration of standard modules.

The essence of typical design method of information systems based on parametric adjustment of AP is to determine the criteria for evaluation of AP, evaluation of many AP applicants according to the formulated criteria and selection of AP with the highest integrated score [7].

Applying the method of typical design of information systems based on the parametric setting of the AP («black box» model) we build a mathematical model of the result of the system to determine the level of quality of diesel fuel (DF) in the fuel tank of the car.

The result of the mathematical model is the information about the quality level of diesel fuel on the consumer's phone screen. The «black box» model uses a sensor that measures fuel quality indicators and compares them with standardized (inputs) and issues a report of the fuel quality level (outputs) [8, 9]. The sensor is a device that allows you to measure the cetane number of fuel and sulfur content. This information is transmitted to the buffer, where the measured values are compared with standard ones. Based on these values, the quality level of the DF is formed, which is transmitted via Bluetooth to the consumer's mobile phone. Consumer sees a message on the phone screen such as: «excellent fuel quality».

Therefore, the inputs of the system are the quality indicators of the DF in the fuel tank of the car *Pi*. The output of the system, that is the result of the system functioning, is the quality level of the DF, which the consumer sees on the screen of his phone. The level of quality is divided into three components: satisfactory quality, good quality and excellent fuel quality.

The system inputs can be represented as:

$$X_i = \lim_{n\to\infty}\frac{1}{n}\sum_{1}^{n} x_i \,. \tag{2}$$

The method of determining of limit and nominal values is used to calculate a sufficient number of inputs for the system. It is based on usage of known maximum permissible values of product quality indicators, which determine the requirements for suitable products or their belonging to a certain level of quality. This method should be used when the limit values of the indicators are defined correctly and justified by the long period of their use. Since the quality indicators of DF are different, we apply the weight parameters to them. For the weighted geometric mean, the weight parameter m_i is determined by the formula:

$$m_i = \frac{\left(\dfrac{1}{\lg\left(P_{inv} - P_{iav}\right)}\right)}{\sum_{i=1}^{n}\left(\dfrac{1}{\lg\left(P_{inv} - P_{iav}\right)}\right)}, \tag{3}$$

where P_{inv} – nominal value of P;

P_{iav} – maximum allowable value of the indicator P.

Then the inputs of the system can be represented as: $X_i = x_i * m_i$ so

$$X_i = \lim_{n\to\infty}\frac{1}{n}\sum_{1}^{n} x_i * \frac{\left(\dfrac{1}{\lg\left(P_{inv} - P_{iav}\right)}\right)}{\sum_{i=1}^{n}\left(\dfrac{1}{\lg\left(P_{inv} - P_{iav}\right)}\right)} \tag{4}$$

To determine the outputs of the system and apply a mixed method of the quality level assessment of product. It is based on the common usage of single and complex indicators and is used when the set of single quality indicators is sufficiently numerous and the analysis of the values for each indicator by the differential method does not allow to obtain generalized conclusions. In the case of a mixed method of quality level assessment of product, it is necessary to: at the first, combine part of the unit indicators into groups and determine the corresponding complex indicator for each group. Important indicators can be used as single; at the second, on the basis of the received set of complex and unit indicators to estimate a quality level of product by a differential method.

In general, basis on complex weighted arithmetic or geometric determinants of the formula for the product quality level by the mixed method can be as follows:

$$K=\sum_{j=1}^{T}\left[A_j*\sum_{i=1}^{H_i}\left(y_i*q_i\right)\right], \tag{5}$$

where T – is the number of groups of quality indicators;

H_i – the number of quality indicators in the *j-th* group;

A_j – parameter of weight of the *j-th* group of quality indicators;

q_i – parameter of weight of the *i-th* group of quality indicators.

Then the outputs of the system can be represented as:

$$Y_i=K=\sum_{j=1}^{T}\left[A_j*\sum_{i=1}^{H_i}\left(y_i*q_i\right)\right]. \tag{6}$$

Based on expression (1), we can write a mathematical model of the system operation result, which was considered on the example of determining the quality level of diesel fuel, in the form:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{1}^{n}x_i*\frac{\left(\dfrac{1}{\lg\left(P_{inv}-P_{iav}\right)}\right)}{\sum_{1}^{n}\left(\dfrac{1}{\lg\left(P_{inv}-P_{iav}\right)}\right)}\to\sum_{j=1}^{T}\left[A_j*\sum_{i=1}^{H_i}\left(y_i*q_i\right)\right]. \tag{7}$$

This mathematical model characterizes the limited choice of system inputs, which are provided with weight parameters, so that you can summarize these inputs and present them as a comprehensive indicator of quality. The output of the system is the level of product quality, which characterizes the fuel quality in one of three components. The model of the result of the system operation is quite complex. It involves determining the level of diesel fuel quality according to the parameters received at the input of the system [10].

Taking into account the need to determine a certain number of inputs to the system, which is represented by the «black box» model, we can consider some set M of these inputs $\{x_i\}$. The quality metrics to be determined from this set will be compared to standardized metrics. As a result, at the output of the system we obtain some set of outputs $\{y_i\}$. Between the inputs and outputs of the system there is feedback, which ensures the correct choice of system parameters (Fig. 4).

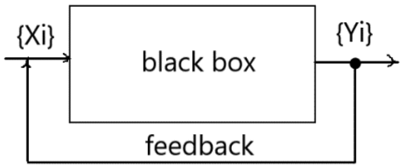

Fig.4. Feedback black box model

Accordingly, the selected quality indicators, which are inputs to the system to determine the quality level, can be represented as functions $f_i(x)$.

In this case, the fundamental question is the measure of similarity between the functions $f_1(x)$ and $f_2(x)$, where $f_1(x)$ – is a function obtained by measuring indicators, $f_2(x)$ – is an ideal function, so standardized indicators are given and which must be approached.

Take two arbitrary functions $\varphi(x)$ and $\psi(x)$, which are given on the set of initial input signals M [11]. At the first stage we consider the case of boiler M has a finite value. Denote by P the number of all elements of the initial set M, and $U(\varphi, \psi)$ is the number of elements with M on which the functions $\varphi(x)$ and $\psi(x)$ differ. The closeness between the functions $\varphi(x)$ and $\psi(x)$ can be represented as:

$$M(\varphi,\psi) = U(\varphi,\psi)/P. \quad (8)$$

Assume that in the next stage appearance of the parameter x of the initial set M is based on the probability distribution $Q(x)$, which represents the probability of the appearance of the parameter x from the set M. In this case, the appearance of one parameter with M does not depend on the appearance of another. The closeness between the functions $\varphi(x)$ and $\psi(x)$ in this case will be determined by:

$$l(\varphi,\psi) = \sum_{x\in M} |\psi(x) - \varphi(x)| * Q(x). \quad (9)$$

In the case when the appearance of x is random, then $|\psi(x) - \varphi(x)|$ also random. Mathematical expectation $M_x|\psi - \varphi|$ quantity $|\psi(x) - \varphi(x)|$ will be equal

$$M_x|\psi - \varphi| = \sum_{x\in M} |\psi(x) - \varphi(x)| * Q(x). \quad (10)$$

So, it follows from (9) and (10) that

$$M_x|\psi - \varphi| = l(\varphi,\psi). \quad (11)$$

In expression (9) it is assumed that $\varphi(x)$ and $\psi(x)$ have a deterministic character. However, when $\varphi(x)$ and $\psi(x)$ are random with respect to x, the closeness between these functions can be represented as

$$\rho(\varphi,\psi)=\sum_{x\in M} M_x(x)*Q(x). \tag{12}$$

Note that the randomness of the functions $\varphi(x)$ and $\psi(x)$ means that for each input signal from the set M a set of functions $\varphi_0(x)$, $\varphi_1(x), \psi_0(x), \psi_1(x)$ (respectively $\varphi_i(x)$ or $\psi_i(x)$, where i=0,1) determines the probability that $\varphi_i(x)$ or $\psi_i(x)$ takes the value i.

Given that $\varphi(x)$ and $\psi(x)$ take values independently of each other, we can say that $|\psi(x)-\varphi(x)|$ takes a value of zero for x with probability

$$X_0(x)=\psi_0(x)*\varphi_0(x)+\psi_1(x)*\varphi_1(x). \tag{13}$$

Similarly, the difference $|\psi(x)-\varphi(x)|$ takes the value of one for x with probability

$$X_1(x)=\psi_0(x)*\varphi_1(x)+\psi_1(x)*\varphi_0(x). \tag{14}$$

Therefore, the following situation will occur:

$$M_x(x)-1+X_1(x)+0*X_0(x)=\psi_0(x)*\varphi_1(x)+\psi_1(x)*\varphi_0(x). \tag{15}$$

Based on expressions (12) and (14), the similarity between the functions can be represented as follows:

$$\rho(\varphi,\psi)=\sum_{x\in M}[\psi_0(x)*\varphi_1(x)+\psi_1(x)*\varphi_0(x)]*Q(x). \tag{16}$$

In the case of a continuous space of the set M, the measures of proximity by expressions (1.8), (1.9), (1.11) can be represented as follows:

$$r(\varphi,\psi)=\sigma[M(\varphi,\psi)]/\sigma M;$$
$$l(\varphi,\psi)=\int_M|\psi(x)-\varphi(x)|*p(x)dx;\ \rho(\varphi,\psi)=\int_M M_x(x)*p(x)dx.$$

In this case, the value of σ specifies some measure for M, and σM represents the measure of the whole set M, and accordingly $\sigma[M(\varphi, \psi)]$ is a measure of the set $M(\varphi, \psi)$. The set $M(\varphi, \psi)$ represents the total set of all objects for which the function ψ is different from the function φ, and $p(x)$ indicates the probability density of the object with plural M.

Research results. Therefore, choosing the necessary functions with a certain probability from the set M, we have specific inputs to the system. By expression (7), which describes the mathematical model, we obtain specific outputs that will characterize the result of the system operation of the presented «black box» model.

The «black box» model is the easiest to obtain the result of the system. Meanwhile, when it is created, various difficulties often arise. They are mainly due to the variety of possible options for establishing links between the object and the environment in which it is located. When using the model, it is necessary to take into account various factors, clearly define the final and initial goals. Execution of the latter is often extremely important to obtain the planned results of observation.

Conclusion. So, the mathematical model of the system operation, considered on the example of the «black box» model, is to determine the quality level of diesel fuel in the car tank, observing only the inputs and outputs, without focusing on the internal structure of the system. This method of system investigation is to determine the quality of diesel fuel, such as cetane number and sulfur content, which are measured directly in the fuel tank and transmit the information to the consumer's phone about the quality level of this fuel. The «black box» model determines the correct definition of the system inputs, which is indicators of diesel fuel, which characterize its quality. The outputs are the result of the system operation and the establishment of the appropriate level of fuel quality. The described mathematical model allows receiving the information on quality of a product which is in a fuel tank of the car. The result of the system operation can be considered as solved if the quality level determined by the sensor in the tank of the car corresponds to the quality passport of this product.

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МАТЕМАТИЧНА МОДЕЛЬ ФУНКЦІОНУВАННЯ СИСТЕМИ ТИПУ "ЧОРНИЙ ЯЩИК"

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В даній статті розглядається математична модель функціонування системи на прикладі моделі «чорний ящик». Входами даної моделі є параметри деякої множини, які потрібно визначити і порівняти з уже існуючими. Виходами служить інформація про стан досліджуваного об'єкта за параметрами з деякої множини, яка може бути представлена у вигляді звітів, графіків, документів. В процесі використання моделі «чорний ящик» не зосереджується значна увага на внутрішній будові системи. Як приклад розглядається математична модель функціонування системи визначення рівня якості дизельного палива у баку автомобіля. Результатом функціонування системи є інформація про відповідний рівень якості палива, яка надходить на мобільний телефон споживача. У процесі роботи системи представляється можливість впливати різним чином на входи системи та аналізувати її реакцію на відповідні вхідні дії. При цьому ступінь реакції вхідних дій принциповим чином пов'язана з різноманітністю станів виходів системи.

Ключові слова: *Математична модель, система, чорний ящик, метод, множина, показники якості, вхід, вихід.*

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